

Solutions to Problem 1.

a.

$$\begin{aligned}\Pr\{3 \leq Y \leq 5\} &= \Pr\{Y = 3\} + \Pr\{Y = 4\} + \Pr\{Y = 5\} \quad (\text{because } Y \text{ only takes on values } 0, 1, 2, \dots) \\ &= \frac{e^{-3}3^3}{3!} + \frac{e^{-3}3^4}{4!} + \frac{e^{-3}3^5}{5!} \approx 0.4929\end{aligned}$$

b.

$$\begin{aligned}\Pr\{3 \leq Y \leq 5\} &= \Pr\{2 < Y \leq 5\} \quad (\text{because } Y \text{ only takes on values } 0, 1, 2, \dots) \\ &= F_Y(5) - F_Y(2) \\ &= \sum_{k=0}^5 \frac{e^{-3}3^k}{k!} - \sum_{k=0}^2 \frac{e^{-3}3^k}{k!} \\ &= \sum_{k=3}^5 \frac{e^{-3}3^k}{k!} \approx 0.4929\end{aligned}$$

c.

$$\begin{aligned}\Pr\{Y > 5\} &= 1 - \Pr\{Y \leq 5\} \quad (\text{because } Y \text{ only takes on values } 0, 1, 2, \dots) \\ &= 1 - F_Y(5) \\ &= 1 - \sum_{k=0}^5 \frac{e^{-3}3^k}{k!} \approx 0.0839\end{aligned}$$

d. $E[Y] = 3$

e. $\text{Var}(Y) = 3$

Solutions to Problem 2.

a.

$$\begin{aligned}\Pr\{2 \leq G \leq 4\} &= \int_2^4 \frac{1}{3} e^{-a/3} da \\ &= \left[-e^{-a/3} \right]_{a=2}^4 \\ &= e^{-2/3} - e^{-4/3} \approx 0.2498\end{aligned}$$

b.

$$\begin{aligned}\Pr\{2 \leq G \leq 4\} &= \Pr\{2 < G \leq 4\} \quad (\text{because } G \text{ is continuous}) \\ &= F_G(4) - F_G(2) \\ &= (1 - e^{-4/3}) - (1 - e^{-2/3}) \\ &= e^{-2/3} - e^{-4/3} \approx 0.2498\end{aligned}$$

c.

$$\begin{aligned}\Pr\{G > 4\} &= 1 - \Pr\{G \leq 4\} \quad (\text{because } G \text{ is continuous}) \\ &= 1 - F_G(4) \\ &= 1 - (1 - e^{-4/3}) \\ &= e^{-4/3} \approx 0.2636\end{aligned}$$

d. $E[G] = \frac{1}{1/3} = 3$

e. $\text{Var}(G) = \frac{1}{(1/3)^2} = 9$

Solutions to Problem 3.

a.

$$\begin{aligned}\Pr\{1 \leq T \leq 2\} &= \Pr\{1 < T \leq 2\} \quad (\text{because } T \text{ is continuous}) \\ &= F_T(2) - F_T(1) \\ &= \left[1 - \sum_{k=0}^3 \frac{e^{-(2/5)(2)} ((2/5)(2))^k}{k!} \right] - \left[1 - \sum_{k=0}^3 \frac{e^{-(2/5)(1)} ((2/5)(1))^k}{k!} \right] \\ &= \sum_{k=0}^3 \frac{e^{-2/5} (2/5)^k}{k!} - \sum_{k=0}^3 \frac{e^{-4/5} (4/5)^k}{k!} \approx 0.0083\end{aligned}$$

b. $\Pr\{T = 2\} = 0$ because T is continuous

c.

$$\begin{aligned}\Pr\{T > 2\} &= 1 - \Pr\{T \leq 2\} \quad (\text{because } T \text{ is continuous}) \\ &= 1 - F_T(2) \\ &= 1 - \left[1 - \sum_{k=0}^3 \frac{e^{-(2/5)(2)} ((2/5)(2))^k}{k!} \right] \\ &= \sum_{k=0}^3 \frac{e^{-4/5} (4/5)^k}{k!} \approx 0.9909\end{aligned}$$

d. $E[T] = \frac{4}{2/5} = 10$

e. $\text{Var}(T) = \frac{4}{(2/5)^2} = 25$